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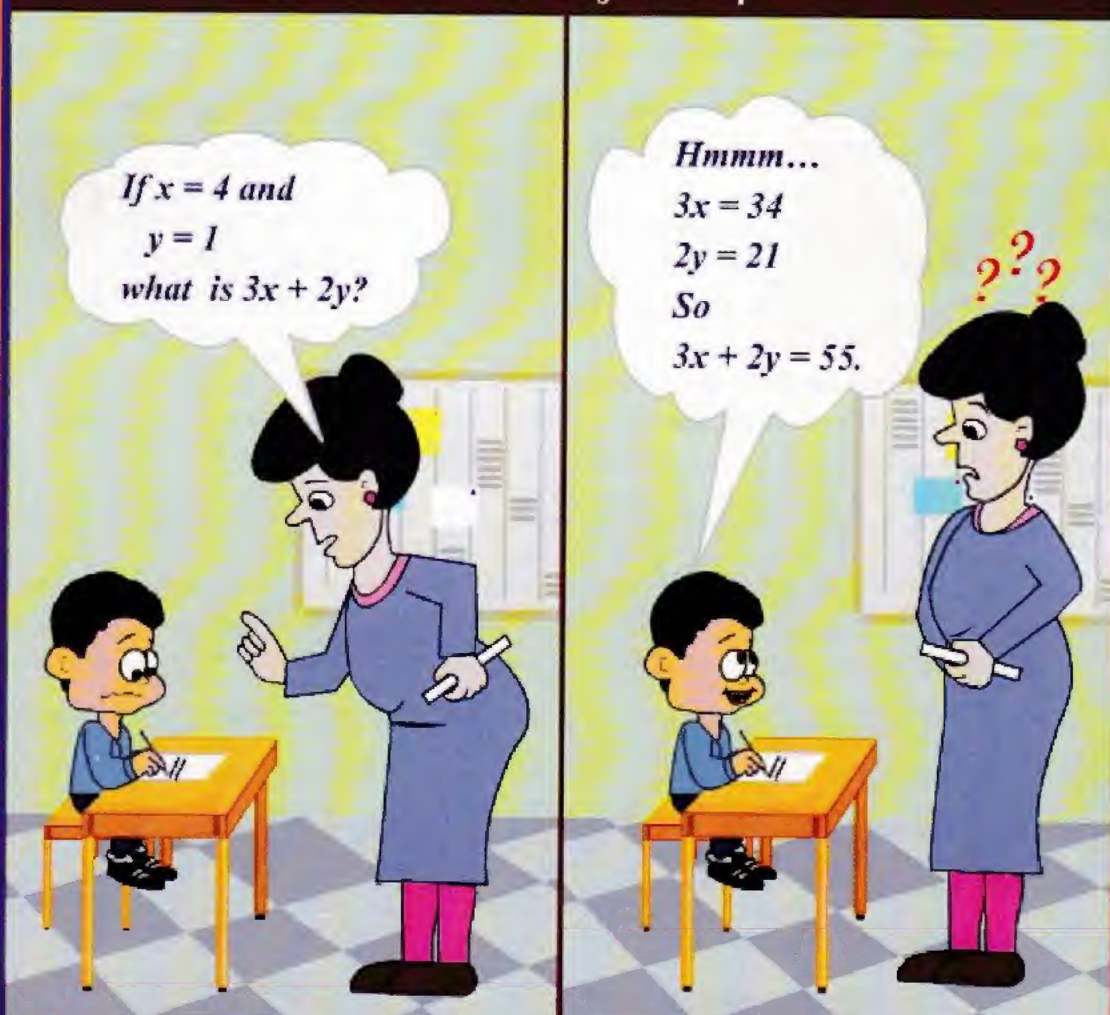
***Editor***

R. ATHMARAMAN


## *The Missing Concept*



# JM

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ınior mathematician (JM) is a Mathematics magazine, principally meant for youngsters ' age between 8 and 16.

- Aims to interact directly with the fresh, young and receptive minds, motivating them in their appreciation and application of Mathematics.
- Aspires to present Mathematics as a lovable subject, satisfying to the serious-minded and pleasurable for others, removing math-phobia.
- Provokes young student-authors to write their own discoveries and creative thoughts.

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***From the desk of the Editor***

## *The need of the hour:* A TINY MATH CIRCLE

This editor fondly remembers one of his former pupils, a naughty but industrious and awfully affectionate class 10 learner, who came one day with an apologetic manifestation and divulged the reason for his remorseful look.

"Sir, all along we students have been believing for sure that we all are exposed to quite a vast curriculum in Mathematics and possess the know-how to solve challenging problems and hence are superior in this respect to our precursors who studied only routine Arithmetic, Algebra and Geometry. We now study enriched mathematics with newer and heavier topics, included day by day. But when I scrutinized the problems asked in the SSLC Course during the beginning of the 20$^{th}$ Century, in an algebra book of my aged uncle, 'A Class book of Algebra', by P.R. Subramania Iyer and attempted casually a few questions, my egotism just got hammered. Here is the book, sir, just peruse it. Does the contemporary syllabus in mathematics make us really shrewder and gifted?" The student submitted an assemblage of loose sheets. [Impressed with the contents, the editor took photo copies of the sheets; it was a very costly affair, in those years! A bound copy of these sheets is now available at the AMTI Library]. A few sample problems are listed elsewhere in this journal. The JMs may like to solve these as a rewarding exercise. Would the present learners be able to confidently face such questions in their school-final examinations? (Do not imagine that the scene is not that bad in colleges and universities!).

The question that the inquisitive student raised is still a hotly discussed subject. Are the present students of mathematics academically proficient to confront varied questions which they might not have seen in their textbooks? There was a time when there were several textbooks, containing wide-ranging problems which the examiners used in their question papers. They also coined original problems. Now, neither the student nor the parent (not to mention politicians who will see an 'evil' design in such tasks) would welcome such a move; the teachers too have lost their zeal. Even the pattern of non-routine questions that are asked in many Olympiad programmes, [thanks to numerous fake National and International Olympiads], are getting routine in some way or other. The result is disastrous. The art and science of problem solving is orphaned callously.

What, then, can JMs do to improve the situation?

While you will do well to take care of your 'knack of scoring' (!) in the tests and examinations, never be contended with the feeble subject matter of the 'stereotyped' questions but go all out in search of challenging problems. (Web sources are handy!). Form a tiny "Math Circle" with four or five like-minded enthusiasts. Enjoy the fun of collecting, dissecting, discussing and solving diverse problems.

A day will dawn when many others would venture upon such "Math Circles".

## SAMPLE PROBLEMS FROM SSLC EXAMINATION OF GOOD OLD DAYS!

1. Prove algebraically: if the product of $x$ and $y$ is constant, $x+y$ is least when $x=y$. Interpret this geometrically as regards the
   i) perimeters of rectangles of given area, and
   ii) lengths of chords in a given circle passing through a given point within it.

   **(SSLC – Year 1911)**
2. Given that $x=2t-1$ and $y+5=24t-16t^2$, express y as a f unction of $x$, and find graphically the value of $x$ for which that function is a maximum and the value for which it vanishes. **(SSLC – Year 1912)**
3. A man purchases some garden land, and reserving five-elevenths of it for his own use, sells the rest in such a manner that the actual cost to him for his own portion is reduced by a half. Had he sold it as Rs.150 more an acre, he would have been a gainer by the whole extent of land which he reserved for himself. At how much an acre did he purchase the land; and at how much did he sell it?

   **(SSLC - Year 1913)**
4. Draw the graph of $y=3-4x-4x^2$. Thence find the roots of the equation $4x^2+4x-3=0$. Show that the expression $3-4x-4x^2$ is positive for all values of x between -1.5 and 0.5 and negative for all real values of x outside these limits. Also find the maximum value of $3-4x-4x^2$.
   Also obtain algebraically the roots of the equation $4x^2-4x-3=0$ and the maximum value of the expression $3-4x-4x^2$. **(SSLC – Year 1916)**
5. A student goes daily from Mylapore to the Law College, a distance of 5 miles. He walks at the rate of 4 miles an hour until a tramcar whose speed is 10 miles an hour overtakes him; then he gets into it and completes the journey. Obtain the equation connecting $x$ the number of minutes occupied by the whole journey with $y$ the number of minutes occupied in walking.

Draw the graph of this equation, specifying the scales neatly on the axis. If on a certain day, the whole journey takes 33 minutes, find the distance he walks on that day. **(SSLC – Year 1918)**

6. In an action between two battleships $A$ and $B$, the ship $A$ fired $2\frac{1}{2}$ times as many shells as $B$. The total number of hits was to the total number of misses as 3:25. The number of $B$'s misses was 390, and $B$'s hits exceeded $A$'s hits by 65. Find the number of shells fired and the number of hits made by each. **(SSLC – Year 1921)**
7. $ABCD$ is a square of side 8". $P$ is a point in the side $AB$, distant $x$" from $A$. Squares are described on $AP$ and $BP$ within the given square. Express in terms of $x$ the area of that portion of the square $ABCD$ which lies outside the squares on $AP$ and $BP$. Tabulate the values of the area as $x$ varies from 0 to 8, and represent the results on a graph.
   Also find the position of $P$ for which the area is greatest. **(SSLC – Year 1922)**

8. Two persons $P$ and $Q$ start at the same time from one extremity $A$ of a diameter $AB$ of a semicircular field $ACB$. $P$ walks along the circumference and $Q$ along the diameter so that they reach $B$ the other extremity of the diameter in $t$ hours after starting. Then $P$ walks along the diameter from $B$ to $A$ and $Q$ along the circumference $BCA$, both inter-changing their paths. How many hours start must $P$ give to $Q$ so that both of them may reach $A$ at the same time? Assume that the radius of the semicircle is $r$ miles and that the ratio of the circumference to the diameter is $\pi$. Frame an equation and solve the problem. What is your remark on the answer? **(SSLC – Year 1911)**
9. Solve graphically, the equations $y = x^2 - x - 2$ and $2x - 3y = 3$, and *check* the result by an **algebraic** solution. **(SSLC – Year 1924)**

10. A rectangular garden is to be divided into six equal rectangular portions by three lines parallel to the sides. If two of the lines are drawn parallel to the length and the remaining one parallel to the breadth of the garden, the perimeter of each portion is 238 yards; if two of the lines are drawn parallel to the breadth and the third parallel to the length, the perimeter of each portion is 232 yards. Find the area of the garden. **(SSLC – Year 1930)**

(*Editor*: Try to solve as many of the above problems as possible. Do you have any short-cuts or interesting connections to be shared with others? Please send them to us.)

# 8 EIGHTS TO GIVE 1000

***Mj. S.A. RAHIM, DIREKTOR, MATHEMAJIK SOSITY, BENGALURU***
***email :*** *mathmaj2013@gmail.com* ***WEBSITE :*** *www.mathmaj.com*

It is fascinating and satisfying to solve a Problem / Puzzle. More thrilling it is to find different solutions to the same Problem / Puzzle and also to invent more Problems / Puzzles inspired by this!

There is a well known Puzzle: "**Give 1000 using 8 times eight**".

A Solution is $888 + 88 + 8 + 8 + 8 = 1000$. Here only 2 operations, namely addition and grouping have been used.

| Usage | Solutions | Example |
|---|---|---|
| +, − , ÷, , only | at least 4 | $\{(8 + 8) \div 8\}\{(8 \times 8 \times 8) - 8\} - 8 = 1000$ |
| grouping also | at least 7 | $(8888 - 888) \div 8$ |
| power & root also | at least 2 | $8(8 + 8)(\sqrt{8})(\sqrt{8}) - 8 - 8 - 8$ |
| factorial also | at least 3 | $8! \div 8[8 - \{(8 + 8 + 8) \div 8\}] - 8$(Note: $8! = 40{,}320$) |
| Decimal also | at least 7 | $8888 \div 8{\cdot}888$ |
| Using Σ also | | $(\Sigma\, 8 \times \Sigma\, 8) - (\sqrt{8} \times \sqrt{8})\,\{\Sigma(\sqrt{8} \times \sqrt{8})\} - (\sqrt{8} \times \sqrt{8})$ |

A Total of 24 Solutions has been found by the students of the author.

### *A simple challenge*

Study the Magic Square given here.

| | | |
|---|---|---|
| **2** | **9** | **4** |
| **7** | **5** | **3** |
| **6** | **1** | **8** |

This arrangement of the first 9 natural numbers adds up to the same total when the sum of the numbers in each row, each column and each diagonal is found. (That is why it is magic!).

Seven other magic squares can be formed from the numbers 1,2,3,4,5,,6,7,8 and 9.

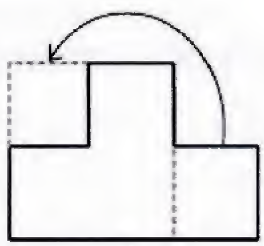
Find t hem.

Do you use the ideas of Reflection, Translation or Rotation?

# *CAN YOU VISUALIZE?*

Ability to visualize often helps in solving problems, particularly those from Geometry. You find hereunder several figures. All that you have to do is to divide each into two parts (not necessarily identical) by one single straight cut with a blade or scissors and place the two pieces together adjacent to each other again so that you get an exact square. Better it would be to take a (traced or photocopied) copy of this sheet, visualize the process and then make a trial.

The solution for the first figure is suggested as a clue. Solutions can be found elsewhere in this issue.



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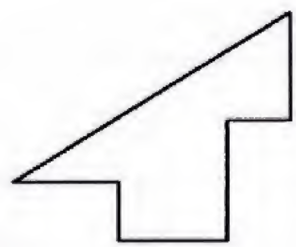

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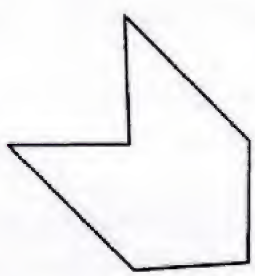

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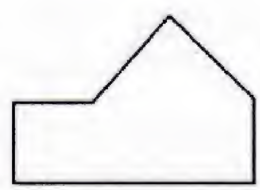

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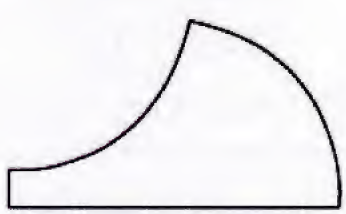

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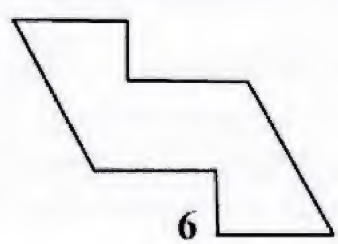


6

# *WHAT IS A CONJECTURE?*

***-S. Padmanaban, Sourashtra HSS, Madurai.***

Very often you might have heard people saying, "I am very much sure that this particular thing is true, but unfortunately I am unable to prove it". Such a kind of guess (a mathematical statement for our discussion), which appears to be true but has not been formally proven is called a mathematical conjecture. It has its basis in inductive reasoning, where you tend to base your judgment on incomplete information. You could be right or wrong and it may take quite a long time to prove or disprove your conjecture or it may remain a conjecture even after several centuries.

Conjectures are not theorems. A conjecture becomes a theorem if and only if it is proved; and just vanishes from the scene when it is disproved. Let us see some examples.

Consider the multiplications of the following 2-digit numbers by 11:

| | | | | | |
|---|---|---|---|---|---|
| 81 | 54 | 43 | 71 | 25 | 62 |
| × 11 | × 11 | × 11 | × 11 | × 11 | × 11 |
| 81 | 54 | 43 | 71 | 25 | 62 |
| 81 | 54 | 43 | 71 | 25 | 62 |
| 891 | 594 | 473 | 781 | 275 | 682 |

You find:

$81 \times 11 = 891$

$54 \times 11 = 594$

Now you are tempted to conjecture that, when you multiply a two digit number by 11, to get the answer, you simply insert the digit 9 in the middle of the digits of the given number to get the answer.

But wait; the next two products are:

$43 \times 11 = 473$

$71 \times 11 = 781$

You observe that your earlier guess does not work here; the middle digit in the answer is not 9. Hence you try to revise your guess; the middle number is after all the sum of the digits of the given number. That is, you conjecture that the middle digit of the product is the sum of the digits of the original number that is multiplied by 11.

Is it over? Have we seen enough examples? Will this conjecture work always? Let us try with a few more multiplications.

$$\begin{array}{r} 76 \\ \times\ \underline{11} \\ 76 \\ \underline{76\ \ } \\ 836 \end{array} \qquad \begin{array}{r} 38 \\ \times\ \underline{11} \\ 38 \\ \underline{38\ \ } \\ 418 \end{array}$$

$76 \times 11 = 836$

$38 \times 11 = 418$

Bad luck! The last conjecture also fails.

This shows that forming a conjecture itself is not an easy task. Perhaps you should have conjectured that it holds true if the sum of the digits of the given number should be less than 10. (Check!) Let us try to prove our revised conjecture, in more general terms, (so that it can become a theorem!)

Let $ab$ be a two digit number, $a$ denoting tens and $b$ representing ones. Thus the numerical value we have is $10a + b$. When you multiply this value by 11, you get

$$\begin{aligned} (10a + b) \times 11 &= 110a + 11b \\ &= (110a + 10a+ + (10b + b) \\ &= 100\,a + 10a + 10b + b \\ &= 100\,a + 10(a + b) + b \end{aligned}$$

Which is a three-digit number whose Hundreds' place is a, tens' place is $(a+b)$ and ones' place is $b$. The middle digit thus is $(a+b)$ which is the sum of the digits of the given number ab. What happens if $a + b \geq 10$?

While it is sometimes very difficult to prove a conjecture, it may equally be difficult to disprove a conjecture. The expression $x^2 + x + 41$ gives odd primes when $x = 0, 1, 2, 3$, etc. Is there a stage when it will fail? As students extend the table, they will see that the patterns continue, but the polynomial yields a composite number when $x = 40$.

An amazing example is that of $1+1141\boldsymbol{n}^2$ (Sowder and Harel 1998). Are there values for $n$ that make this expression a perfect square? Surprisingly, all $\boldsymbol{n}$ from 1 to 30,693,385,322,765,657,197,397,207 fail to produce a perfect square. You find

$$1141\boldsymbol{n}^2 + 1 = 1036782394157223963237125215^2$$

when $n = 30{,}693{,}385{,}322{,}765{,}657{,}197{,}397{,}208$.

You will now appreciate why mathematicians insist on a proof; any number of examples cannot equal a proof.

## *NUMBER WONDERS*

***T. Dharmarajan, 2/5/4, Teachers' Colony, Coimbatore -641 022.***

Mathematics is enticing for its own beauty. There is a lot to admire mathematical results—truly delightful and arithmetically clever processes. We see a few here. Perhaps some of you are already familiar to JMs, but still remains a fancy to go through again.

### A surprising 5-cycle Number pattern

Take any two positive integers. Get a third number which is the quotient obtained when the second number added to 1 is divided by the first number. Repeat the process then taking the previous second number as first number and the quotient obtained as the second number. What do you find at the end of the 5$^{th}$ step?

As an example, let us take the first number to be 7 and the second number to be, say, 11.

| Step | 1$^{st}$ number | 2$^{nd}$ n umber | Division Process | Quotient |
|---|---|---|---|---|
| I | 7 | 11 | $\frac{11+1}{7}$ | $\frac{12}{7}$ |
| II | 11 | $\frac{12}{7}$ | $\frac{(\frac{12}{7})+1}{11}$ | $\frac{19}{77}$ |
| III | $\frac{12}{7}$ | $\frac{19}{77}$ | $\frac{(\frac{19}{77})+1}{(\frac{12}{7})} = \frac{96}{77} \times \frac{7}{12}$ | $\frac{8}{11}$ |
| IV | $\frac{19}{77}$ | $\frac{8}{11}$ | $\frac{(\frac{8}{11})+1}{(\frac{19}{77})} = \frac{19}{11} \times \frac{77}{19}$ | 7 |
| V | $\frac{8}{11}$ | 7 | $\frac{7+1}{(\frac{8}{11})} = 8 \times \frac{11}{8}$ | 11 |

The cycle repeats! Take any other couple of natural numbers; perform the operations step by step, you get the reappearance of the cycle.

How does this happen? This is where JMs should investigate. Elementary algebraic generalization comes handy.

Let the two natural numbers be '***a***' and '***b***'. Here are the steps that we perform.

| Step | 1st number | 2nd number | Division Process | Quotient |
|---|---|---|---|---|
| I | $a$ | ***B*** | $\frac{b+1}{a}$ | $\frac{b+1}{a}$ |
| II | $b$ | $\frac{b+1}{a}$ | $\frac{(\frac{b+1}{a})+1}{b}$ | $\frac{a+b+1}{ab}$ |
| III | $\frac{b+1}{a}$ | $\frac{a+b+1}{ab}$ | $\frac{(\frac{a+b+1}{ab})+1}{(\frac{b+1}{a})} = \frac{(a+1)(b+1)}{b(b+1)}$ | $\frac{a+1}{b}$ |
| IV | $\frac{a+b+1}{ab}$ | $\frac{a+1}{b}$ | $\frac{(\frac{a+1}{b})+1}{(\frac{a+b+1}{ab})} =$ $\frac{(a+b+1)}{b} \times \frac{ab}{(a+b+1)}$ | ***A*** |
| V | $\frac{a+1}{b}$ | ***A*** | $\frac{a+1}{(\frac{a+1}{b})} = (a+1) \times \frac{b}{(a+1)}$ | ***B*** |

## The number 2519

The number 2519 seems to fall consistently short of the mark when it is subjected to division by the positive integers from 2 to 10.

Here are the results about remainders when 2519 is divided by 2, 3, 4, ..., 10.

| Division of 2519 by | ÷2 | ÷3 | ÷4 | ÷5 | ÷6 | ÷7 | ÷8 | ÷9 | ÷10 |
|---|---|---|---|---|---|---|---|---|---|
| Remainder | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

It is further interesting to note that it is the smallest such number.

There are a few more things. too.
It cannot be written as a sum of three squares.
It is the sum of the first 41 semiprimes (semiprime = product of two primes)
It is a number $n$ such that $2n + 1$, $3n + 2$, and $4n + 3$ are all primes.

## Fascinating fractions

The digits 1, 2, 3, ..., 9 can be arranged to form a pair of natural numbers whose ratio is $\frac{1}{2}$, as follows: $\frac{7329}{14658}$.
This is interesting in itself, but even more fascinating is the fact that the nine digits can also be arranged to form numbers whose ratios are each $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$ and $\frac{1}{9}$.

Here you find them:

| | |
|---|---|
| $\frac{1}{2}$ | $= \frac{7293}{14586} = \frac{7269}{14538}$ |
| $\frac{1}{3}$ | $= \frac{5823}{17469} = \frac{5832}{17496}$ |
| $\frac{1}{4}$ | $= \frac{7956}{31824} = \frac{5796}{23184} = \frac{3942}{15768} = \frac{4392}{17568}$ |
| $\frac{1}{5}$ | $= \frac{2697}{13485}$ |
| $\frac{1}{6}$ | $= \frac{2943}{17658} = \frac{4653}{27918} = \frac{5697}{34182}$ |
| $\frac{1}{7}$ | $= \frac{2394}{16758} = \frac{2637}{18459} = \frac{4527}{31689}$ |
| $\frac{1}{8}$ | $= \frac{3187}{25496} = \frac{4589}{36712} = \frac{4591}{36728} = \frac{6789}{54312}$ |
| $\frac{1}{9}$ | $= \frac{6381}{57429} = \frac{6471}{58239}$ |

## One more!

In the very first issue of Junior Mathematician, (Late) Sri P.K. Srinivasan had given the following problem:

**Study the result $\sqrt{2\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$. Is it true? If it is true, is it a unique case? If it is not unique find others.**

First let us verify the result.

$$\text{RHS} = \sqrt{2\frac{2}{3}}$$

$$= \sqrt{2+\frac{2}{3}}.$$

$$= \sqrt{\frac{8}{3}} = \sqrt{\frac{4\times 2}{3}} = \sqrt{4}\times\sqrt{\frac{2}{3}} = 2\times\sqrt{\frac{2}{3}} = 2\sqrt{\frac{2}{3}} = \text{LHS}$$

So far it is only routine manipulation. But how about getting more such peculiarities?

Again, algebra can assist us.

If we have $\boldsymbol{x}$ in the place of 2 and $\boldsymbol{y}$ in the place of $\frac{2}{3}$, then we have to examine the

equality $\sqrt{x+y} = x\sqrt{y}\,.\sqrt[3]{x+y}$

Squaring both sides, we get $\quad x + y = x^2y \quad or \quad y = \dfrac{x}{x^2-1}$ .

You now have several possibilities:

| Value for $x$ | 2 | 3 | 4 | 5 | . | . | . | . |
|---|---|---|---|---|---|---|---|---|
| Value for $y = \dfrac{x}{x^2-1}$ | $\frac{2}{3}$ | $\frac{3}{8}$ | $\frac{4}{15}$ | $\frac{5}{24}$ | . | . | . | . |

Thus you get many solutions:

$$\sqrt{2\frac{2}{3}} = 2\sqrt{\frac{2}{3}},\quad \sqrt{3\frac{3}{8}} = 3\sqrt{\frac{3}{8}},\quad \sqrt{4\frac{4}{15}} = 4\sqrt{\frac{4}{15}},\quad \sqrt{5\frac{5}{24}} = 5\sqrt{\frac{5}{24}}, \text{etc.}$$

JMs would not stop with this. Let us go further investigating. We saw the case of square root equality. What will happen in the case of cube roots, fourth roots etc.?

Let us probe the case of $\sqrt[3]{x+y} = \boldsymbol{x}\sqrt[3]{y}$

Cubing both sides, we obtain $x + y = x^3y$ and hence $y = \frac{x}{x^3 - 1}$.

As before, calculating $y$ for values of $x$, you get many solutions:

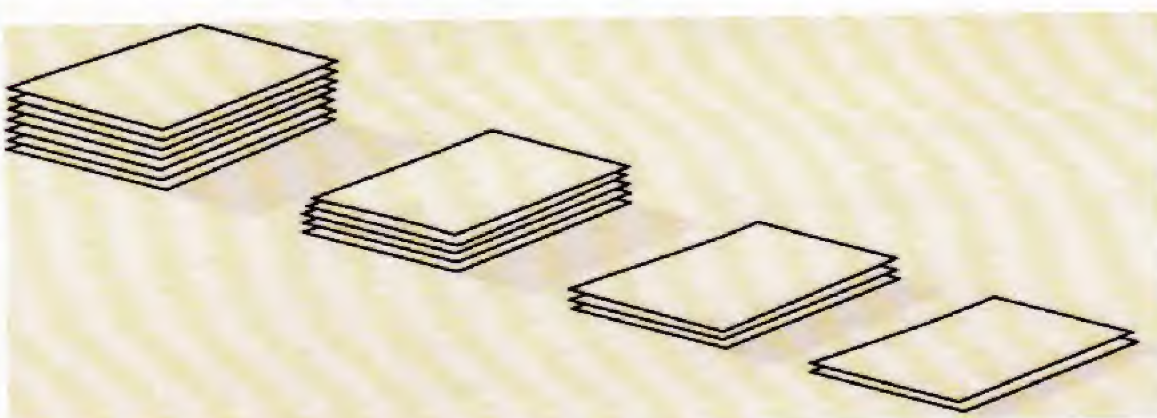
If $x = 2$, $y = \frac{2}{7}$, then $\sqrt[3]{2\frac{2}{7}} = 2.\sqrt[3]{\frac{2}{7}}$

If $x = 3$, $y = \frac{3}{26}$ and $\sqrt[3]{3\frac{3}{26}} = 3.\sqrt[3]{\frac{3}{26}}$.

Can you extend to this to the general case of '$n$-th root'?

## Paper-tearing:

Take a sheet of paper, say a news paper. Tear it in half and put the two pieces one over the other. Now, tear the twin pieces in half and put them one over the other. Repeat this process, again and again, to form a pile of pieces. the height of a piece of folded paper would double in thickness each time it was folded. Suppose you do this 40 times, how high do you think your pile would be at the end?



Suppose you had started with the thickness of paper being 0.01 cm. After 40 tearings, the pile would have $2^{40}$ pieces in it. This amounts to $0.01 \times 2^{40}$ cm high. You find $2^{40}$ is nearly $1.1 \times 10^{12}$. Thus the height of the pile of pieces would be about $0.01 \times 1.1 \times 10^{12} = 1.1 \times 10^{8}\ m$ which means 110,000 *km*! It is a different question if we would be able to tear paper in this way manually! There are many interesting references to this on the web.

***A Gem among mathematicians:***

## *SUBBIAH SIVASANKARANARAYANA PILLAI*
## (S.S.PILLAI)

S.S. Pillai was born on April 5, 1901 in a village named Vallam near Sengottai. His mother Gomathi passed away when Pillai was 12 years old. Pillai completed his primary education in Illathur. During Pillai's primary education his father Subbiah passed away but Pillai's teacher Sastri helped him continue his education. After completing his high school education in S.M.S.S Govt High School in Sengottai, Pillai did his Intermediate course in Nagercoil's Christian College and then, he completed B.A in Maharaja College Tiruvananthapuram..

**Born** : April 5, 1901 (Tamilnadu)
**Died** : 31 August 1950 (Cairo, Egypt)
Fame : Pillai's conjecture,
Prillai's Arithmetical function
Pillai's Prime

S.S.Pillai aspired to be a research student in Madras University. Only a first class passed candidate would be admitted for the research study but Pillai had passed in second class. [ Even the math genius Ramanujan faced a similar complexity. A visionary Gilbert D. Walker (Weather Forecast Department Head) and a group of reputed Mathematicians requested the University of Madras to support Ramanujan financially for his research. Representatives Committee of University of Madras recommended to offer Rs 75/- as scholarship every month to Ramanujan with certain terms and conditions. But a few disputed that to take up research and avail the scholarship one has to be a post graduate, while others argued that there are certain ways with which the problem can be solved. There was scope for the approval and signature of the governor to sanction the scholarship. With the Governor's signature, the University offered Ramanujan the scholarship from 1-5-1913 onwards.

The head of Association of Mathematicians, Mr. Hanumantha Rao stood by Ramanujan in times of his struggle. Similarly the Principal of Pachaiappan College, Mr. Chinnathambi Pillai too supported S.S.Pillai tellingothers, "Look at the talent. Do not repeat the same mistake as previously done to Ramanujan. Do not thrust rules on Geniuses". All these yielded good results; the rules were amended. And S.S.Pillai

was admitted as a research student in University of Madras. Under the guidance of Professor Ananda Rao, after four years of research, Pillai received the Honorary DSC degree. This achievement made Pillai the first mathematics student to receive an honorary doctorate in science in the history of University of Madras. Professor Vaidyanadhaswamy too had guided Pillai through Pillai's doctorate course. After receiving the doctorate, in 1929 Pillai joined Annamalai University as a lecturer. He started his research on Number Theory.

After spending seven years on Number theory, on 10-02-1936 Pillai released his findings on for Waring's conjecture. He also released a book confirming the concepts he had talked about. Doctor Pillai's name and fame spread all over the mathematics world. He was invit4ed by many a forein forums to discuss further about his work. But Pillai refused, saying that his own motherland would be enough for his research.

Dr. Pillai was personally invited by Einstein to discuss his research. He was also convinced by many intellectuals to preside over the International Congress of Mathematics and then take up joint research along with Dr Einstein at Briston University. S.S. Pillai who accepted to participate in the conference in 1950 started his journey on August 30 in a flight named *"Star of my land".* The flight landed in Cairo to fill in up the fuel and took off to San Francisco. But unfortunately while flying above the Sahara Dessert, on 31st August at 3.00 am, the flight caught fire which led to a fatal accident. India's dreams were burnt to ashes. Before the end came, Dr Pillai had already published 76 research articles.

## His contributions to mathematics:

He worked mostly in analytical number theory. One of his famous contributions is towards solving Waring's Problem. Dr Pillai worked on Diophantine approximations too and proved many beautiful related results. His proof for Bertrand's postulate was quite fresh and new. Dr Pillai stated several conjectures on perfect powers. This is one among them:

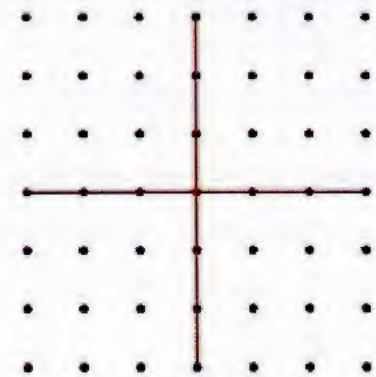
Arrange all the powers of integers like squares, cubes etc. in increasing order as follows:

1, 4, 8, 9, 16, 25, 27, 32, 36, 49, 64, 81, 100, 121, 125, 128,... Let $a_n$ be the $n$-th member of this series so that $a_1 = 1$, $a_2 = 4$, $a_3 = 8$, $a_4 = 9$, etc. Then

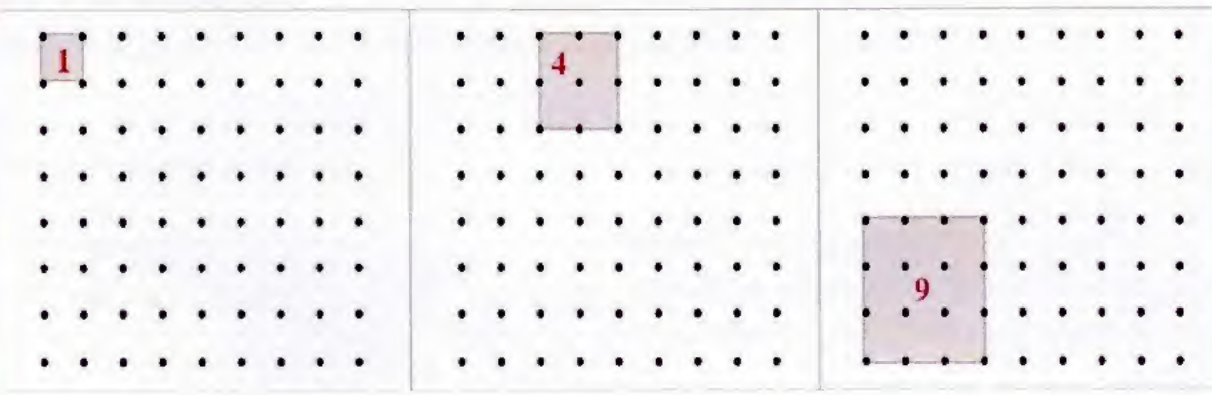
$$\liminf_{n \to \infty} (a_n - a_{n-1}) = \infty.$$

# *DRAWING A SQUARE ON A SQUARE LATTICE*

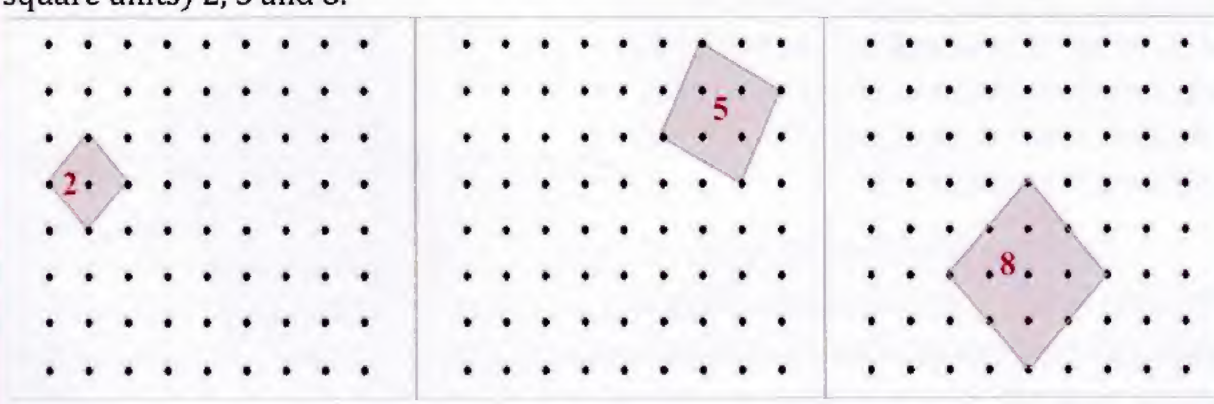
A regularly spaced array of points in a place is usually called a ***point lattice.*** You can draw in the plane point lattices having unit cells in the shape of a square array. Such a lattice has points with coordinates that are integers. (In everyday life, we usually know such an arrangement as a ***grid*** or ***mesh.*** Let us delve into an important problem involving such a lattice.



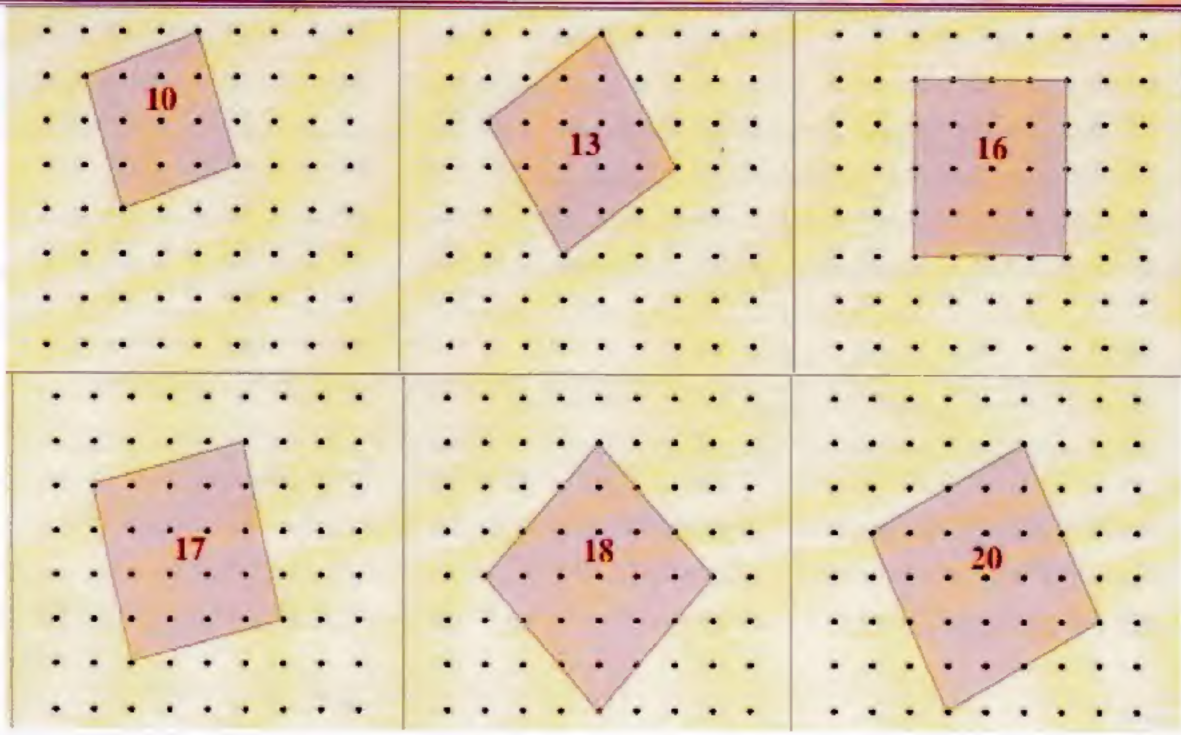
You can certainly draw a square of unit area (that is area = 1 sq.unit) on a square lattice. It should be also easy to construct squares of area 4 units or 9 units.



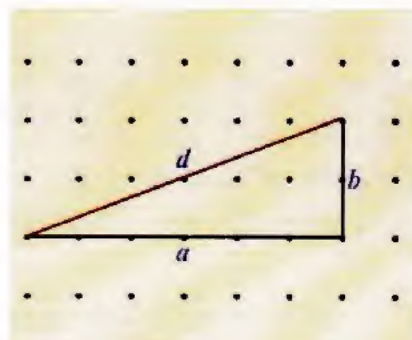

With a little more visualization, you can also draw squares with areas (in square units) 2, 5 and 8.




With some more alertness, you may also erect squares of areas 10, 13, 16, 17, 18 and 20.

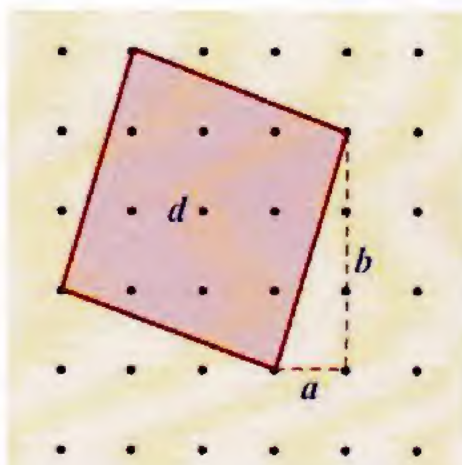



Can a square lattice have area 3 ?




Let us try to take a general number, say, $\boldsymbol{d}$ for area and find out the condition under which it could be the area of a lattice square.

Recall that, by Pythagoras theorem, if $\boldsymbol{d}$ is the length of a line segment joining two points on a square lattice, then it the square root of the sum of two squares of integers $\boldsymbol{a}$ and $\boldsymbol{b}$. (In short, $\boldsymbol{d^2 = a^2 + b^2}$, where $\boldsymbol{a}$, $\boldsymbol{b}$ are integers).




Thus, if $\boldsymbol{d}$ is the area of a lattice square, then the length of its side is $\sqrt{\boldsymbol{d}}$ and from the converse point of view, if you have $(\sqrt{d})^2 = a^2 + b^2$ for some integers $\boldsymbol{a}$ and $\boldsymbol{b}$, then you can construct a square of area $d$ as shown in the figure.

Observe that a lattice square of an area $d$ is possible when (and only when) $d$ is the sum of two square numbers.

You cannot expect a square lattice of area 3 to exist, since no two integers are available the sum of whose squares is 3.

Can we have a square lattice of area 61? It is possible, since $61 = 5^2 + 6^2$. Try to draw it as a rewarding exercise.

There is one more interesting spectacle. If you have two lattice squares of areas, say, $\boldsymbol{x}$ and $\boldsymbol{y}$, then it is possible to construct a lattice square of area $\boldsymbol{xy}$.

This follows from the simple algebra: If $\boldsymbol{x} = a^2 + b^2$ and $\boldsymbol{y} = c^2 + d^2$, then

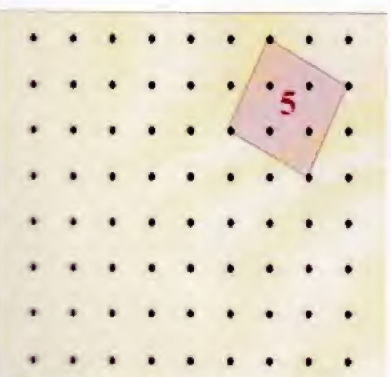
$$\boldsymbol{xy} = (a^2 + b^2)(c^2 + d^2)$$

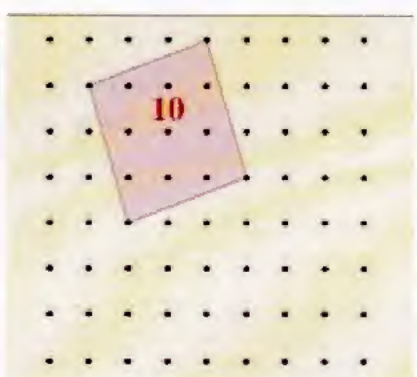
$$= (\boldsymbol{ac} + \boldsymbol{bd})^2 + (\boldsymbol{ad} - \boldsymbol{bc})^2 \text{, a sum of two squares.}$$

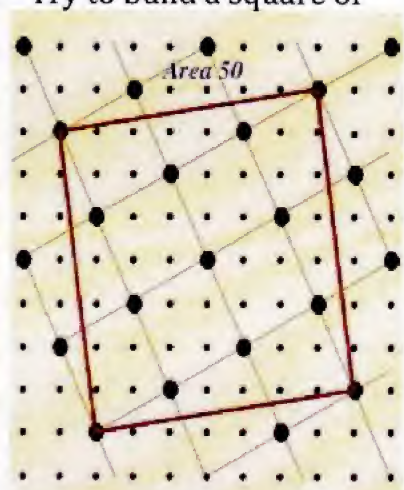
(This is sometimes called '*Brahmagupta-Fibonacci Identity*')

For example, suppose you want to construct a square lattice of area 50. Then all that you need to consider are lattice squares of area 5 and 10 (since $5 \times 10 = 50$).

Create the smaller square (of area 5) as a lattice square. Try to build a square of area 10 on this lattice.




Investigate if this approach works for any pair of lattice square areas $\boldsymbol{x}$ and $\boldsymbol{y}$.

## *A challenge!*

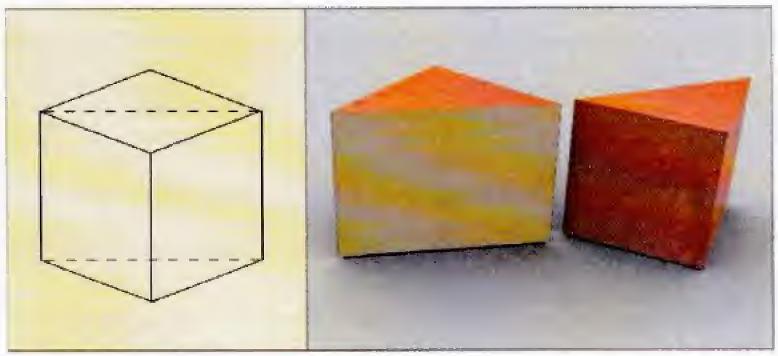
We observed that a lattice square of area $\boldsymbol{d}$ exists iff $\boldsymbol{d}$ is a sum of squares of two natural numbers. Can you conjecture now that "$\boldsymbol{d}$ can be written as a sum of squares of two natural numbers (not necessarily distinct) in a unique way" ? Prove or disprove it.

(Try yourself before turning to solution given elsewhere in this issue).

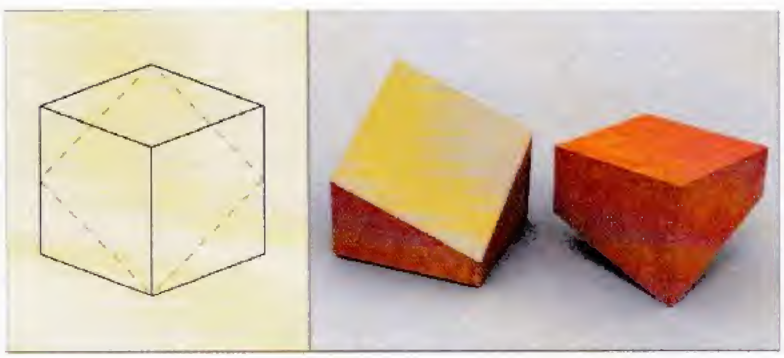
# CROSS-SECTION WITH ONE SLICE

The following is a good learning activity when you have some leisure. It demands your imagination and provokes you to think of alternate possibilities. You may use plasticine making a cube or a vegetable like potato duly diced. Be careful when you use a sharp knife for dissection.

1) A rectangular croo-section:

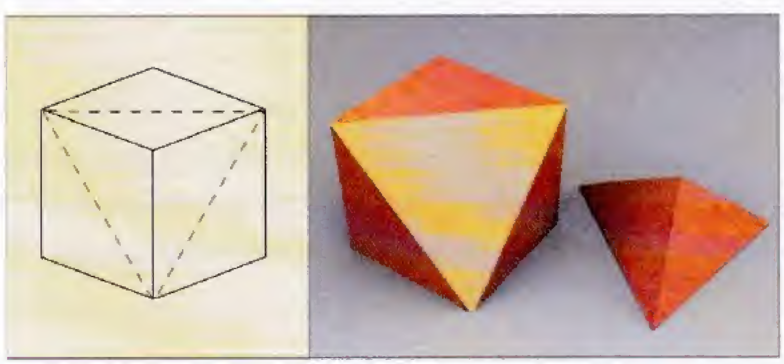


2) A rhombus cross-section:

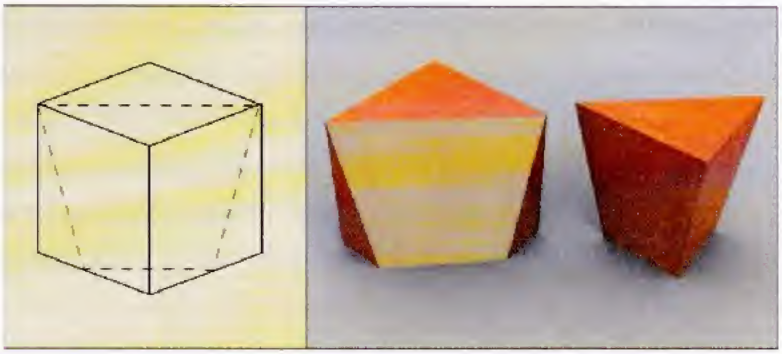


Is the side of the rhombus longer than the edge of the cube?

3) An equilateral triangle cross-section:

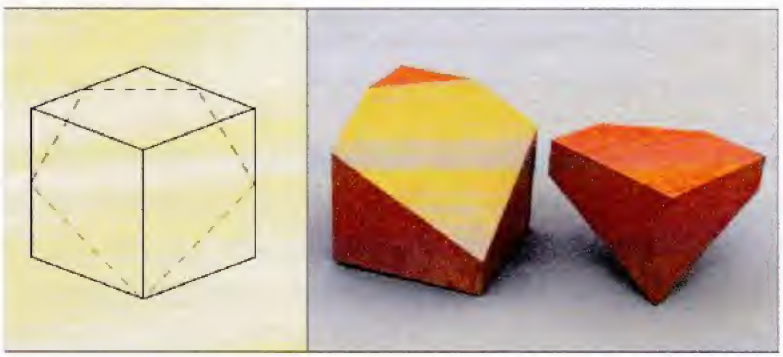


4) A trapezium cross- section:



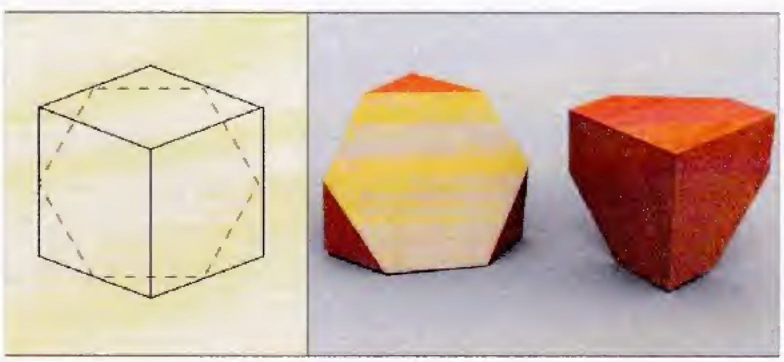
Where does the knife pass through the bottom of the cube?

5) A pentagonal cross-section:



Is it possible to have a regular pentagonal section?

6) A hexagonal cross-section:



Can the hexagonal shape be irregular?

Think about solutions other than given here. That would be the true spirit of a Junior Mathematician.

# *A TASTE OF REAL-LIFE MATHEMATICS*

There is a general complaint that all the mathematics you study is of not much use. There are simple occasions when we use many concepts that we learn in the classroom.

- Suppose your family plans to visit a temple about 80 kms away from your home and you have only 6 hours for the travel time. You want to book a Call Taxi and try to gather availability from two different companies, Bheema taxi and Hanuman taxi.

Bheema travels charges Rs.100 for the first 4 *kms* and thereafter Rs.12 for every *km.* and also charges Rs.40 every extra hour after 5 hours.

Hanuman taxi charges Rs.50 for the first 3 *kms* and thereafter Rs.15 for every *km.* and also charges Rs.30 every hour extra aftr 6 hours.

Which taxi would be economical for travel? Do you use the concepts of direct proportion and unitary method here?

- Flywing airways books tickets for its flight to Delhi. The maximum accommodation in the plane is 320. All the seats are booked. There is however a risk of a few seats getting vacant. This is because, according to calculation of traffic manager, the probability of passengers not missing the flight is 0.96. How many seats can the airline management expect to be empty? Shuld the airline book more than 320 seats in anticipation of absentees?

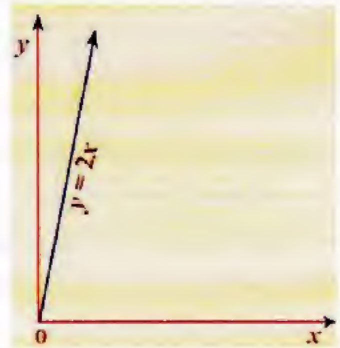
- It is not easy scheduling games in a competition. Tournaments can be on Single Elimination method or Round Robin method. We use a lot of combinatorial ideas to work out the rounds, elimination and places.

- You want to buy a new readymade shirt and a pair of shoes for your birthday. The shopkeeper suggests you a few. How can you be sure that the items suggested will fit you properly? Will the shirt be tight or could the shoes be quite wobbly? Does any mathematical concept help you? (Remember Mean, Median and Mode!)

- From figuring out the amount needed for a party to calculating the tax exemption you want to get, you need mathematics. (You need mathematics even in generation of numbers for your Debit Card!)

# *UNDERSTANDING 'SLOPE' OF A LINE*

***-R. Nandhini, Sri Sarada Secondary School, Chennai-600 086.***

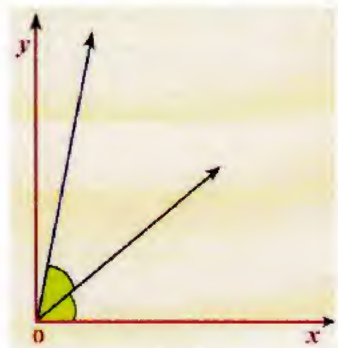



(a)

The idea of the 'slope' of a line is often not well understood. Take the coordinate axes ***Ox*** and ***Oy***. Draw line (say, $\overline{OA}$) whose slope is 2. How will you go about it?

Slope of a line is simply 'rise over run' where rise means the change in ***y*** (going up or down) and run means the change in ***x***, (going from left to right). To find the slope of a line the following steps help:

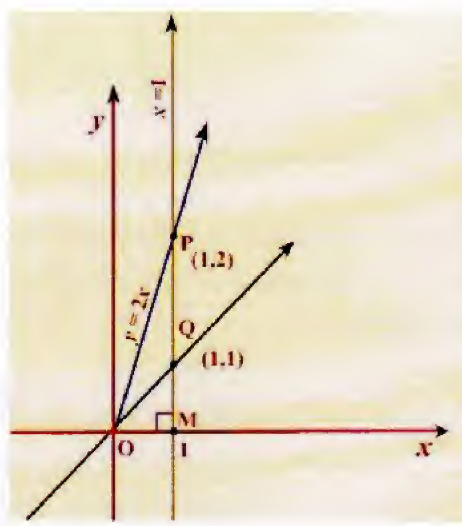
- Take two points on the line.
- Count the rise; Count the run.
- Find slope $= \dfrac{rise}{run}$.




(b)

Now the question is how will you draw a line (through the origin) whose slope is half the slope of the already drawn line $\overline{OA}$ (Fig.a). Since the slope of $\overline{OA}$ is 2, we would need a line with a slope 1.

One would commonly find a student drawing a line through the origin, by just halving the angle between the $\overline{OA}$ and $\overline{Ox}$ (Fig.b) . This process halves the angle but need not halve the ***slope*** $\overline{Ox}$ .



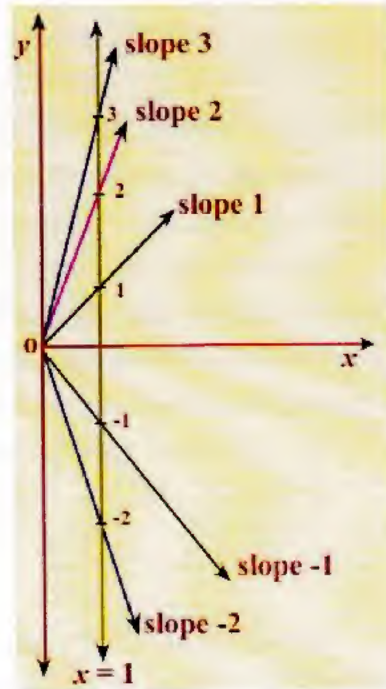

We students discussed about this problem among ourselves. We found in a very old journal an idea similar to what is given here, to remove the misconcpetions that haunt us.

When fig (a) is given, draw the graph of the line ***x*** = 1. It will cut the given line $\overline{OA}$ at a point P. Imagine a segment $\overline{PM}$ perpendicular to ***x*** axis. Since P now has coordinates (OM, PM), you find that PM = 2.

You can now identify another point, say Q, on $\overline{PM}$ , such tnat QM = 1. This leads to the fact

that $\overline{OQ}$ is the line with slope 1. By using a protractor, one can see that $\overline{OQ}$ is not the bisector of $\angle POM$.

The adjacent figure shows how the line x = 1 intersects the lines having different slopes while passing through (1,0).




Observe the pair of lines with slopes 1 and -1 or with slopes 2 and -2. Incidentally we obtain an idea of reflection that can sometimes help to identify lines with certain slopes easily.

This kind of approach can be extended further, for example, to give several lines in a coordinate plane, estimate their slope by eye and then use a technique similar to the above to see how close the estimate is.

## Solving through Visualization!

Consider a square. (fig.1) Join the the veritices of the square to the mid points of its sides, in a systematic way. (fig.2)

Fig. 1

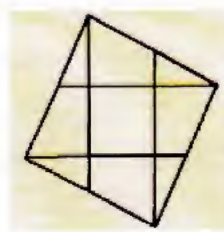
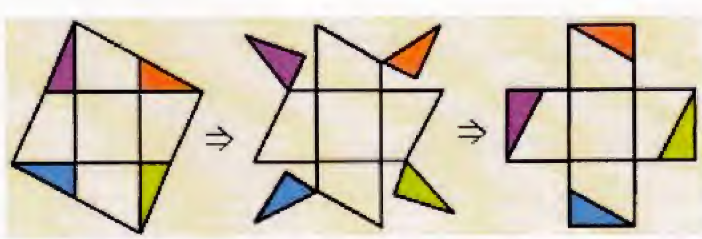


Fig. 2

How will you find a relationship between the area of the square at the centre and the area of the square originally given? Here is one method using a rearrangement of the figure.



Is the area of the Central square, one-fifth of the original square?

## ONE QUESTION – MANY LINES OF ATTACK

***Sumedha D, Class XI, Chettinad Harishri Vidyalayam, Chennai.***

Recently, a question asked in AMTI – NMTC – Junior Level II (31-10-2015) helped me recollect the ideas associated to Geometry, Coordinate Geometry, Trigonometry and Vectors to solve just a single problem.

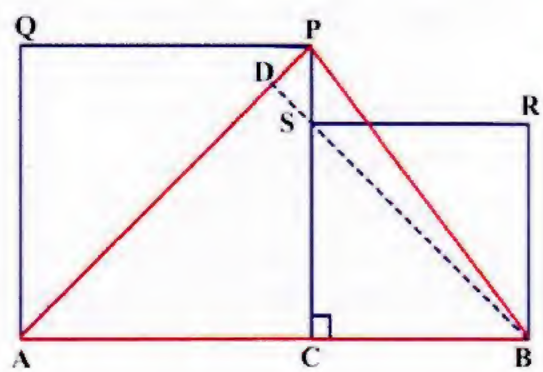
The question was:

$1(b)$. ***AB is a line segment C is a point on AB. ACPQ and CBRS are squares drawn on the same side of AB. Prove that S is the orthocentre of the triangle APB.***

It is sufficient to take any random point on ***AB*** (other than $A$ or $B$ to avoid degenerated triangle ***APB***) to prove it, as it would be applicable to every point on ***AB*** including the mid point of ***AB***.

As we know that the altitudes of a triangle are concurrent, it is enough to show that two altitudes of *ïAPB* pass through $S$, to prove $S$ is the orthocentre of *ïAPB.*

I. Let us construct line segment $\overline{BSD}$, meeting $AP$ at $D$. Since $ACPQ$ is a square, $\angle ACP = 90^o$. So $PC \perp AB$ and hence $PC$ is an altitude of *ïAPB* passing through $S$.




In *ïADB,* $AP$ the diagonal of the square $ACPQ$ bisects $\angle CAQ = 90^o$. Hence

$$\angle BAD = \angle CAD = 45^o \quad ......(i)$$

In *ïADB,* $BS$ the diagonal of the square $CBRS$ bisects $\angle CBR = 90^o$. Hence

$$\angle ABD = \angle CBS = 45^o \quad ......(ii)$$

By angle-sum property of *ïABD*

$\angle ADB = 180^o - (\angle DAB + \angle ABD) = 180^o - (45^o + 45^o) = 90^o$.

$\therefore$ $BD \perp AD$, which gives $BD \perp AP$.

So $BD$ is an altitude of *ïAPB,* passing through S.

Altitudes $PC$ and $BD$ pass through $S \Rightarrow S$ is the orthocentre of *ïAPB.*

II Since $ACPQ$ is a square, $\angle ACP = 90^o$. So $PC \perp AB$ and hence $PC$ is an altitude of *ïAPB* passing through $S$.

BS and RC are diagonals of square CBRS and therefore,

$BS \perp RC \Rightarrow BD \perp RC$.

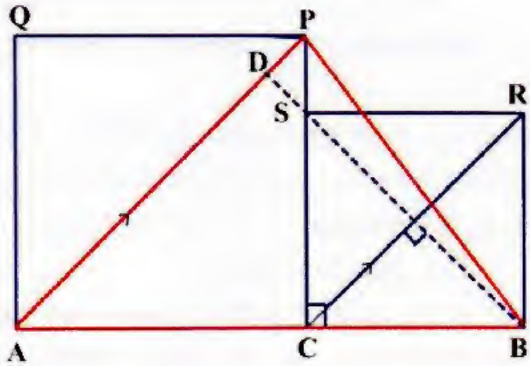
Since AP and RC are diagonals of squares ACPQ and CBRS respectively,

$\angle PAB = \angle PAC = 45^o = \angle RCB$

and so AP // RC. This $\Rightarrow$ $BD \perp AP$.

Thus BD is an altitude of $\Delta$APB.

As altitudes PC and BD pass through S, certainly S is the orthocentre of $\Delta$APB!

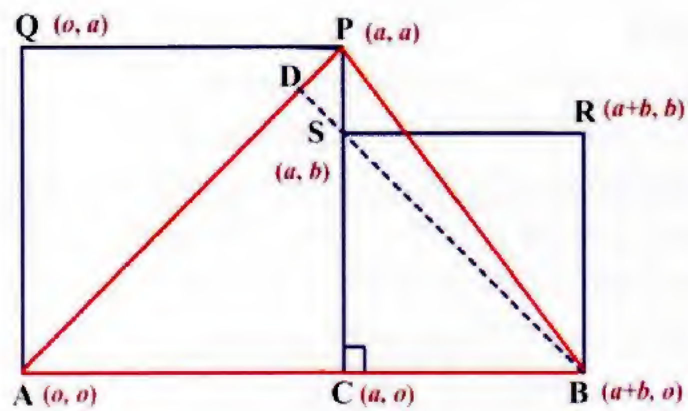



**III** Let us suppose that $AC = a$ units and $BC = b$ units where a, b are positive real numbers. We can now construct coordinates of the points A, C, P, Q, B, R and S as in the figure. We take A as origin and AB lies on positive x-direction.

As $\angle PCA = 90^o$, $PC \perp AB$. That is, PC is an altitude of $\Delta$APB passing through S.

Now it is sufficient to prove that the line containing AP is perpendicular to the line containing BD or BS, so that one can say $BD \perp AP$.

As we know that 'product of slopes of two lines is -1 $\Rightarrow$ they are perpendicular to one another', we try to find the product of the slopes of the lines AP and BS. (We make use of the formula to compute slope of a line joining two points).



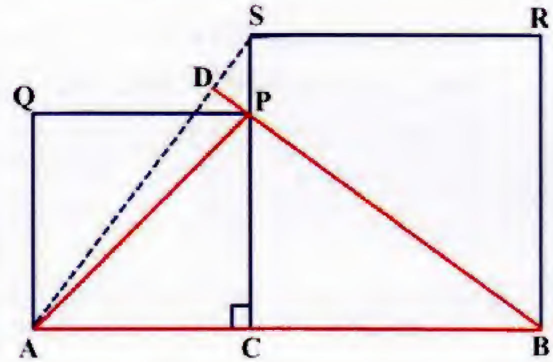

$$(\text{Slope of } \overline{AP}) \times (\text{Slope of } \overline{BS}) = \left(\frac{a-0}{a-0}\right) \times \left(\frac{0 \quad - \quad b}{a+b \quad - \quad a}\right) = \frac{a}{a} \times \frac{-b}{b} = 1 \times (-1) = -1.$$

Therefore, $BD \perp AP$. That is, BD is an altitude of $\Delta$APB, passing through S. Altitudes PC and BD passing through S implies that S is the orthocentre of $\Delta$APB.

**IV.** For this proof, we make a construction. Join AS and then extend BP to meet AS at D. Since $\angle PCB = \angle SCB = 90^o$ we now have $CS \perp AB$.

Thus CS is just an extension of PC.

Consider triangles ACS and PCB.
We find
AC = PC (sides of square ACPQ)
$\angle ACS = \angle ACP = 90^o = \angle SCB = \angle PCB$
CS = CB (sides of square CBRS).
$\therefore$ By SAS congruency property,
$\Delta$ ACS $\equiv$ $\Delta$PCB.
Being congruent parts of these two congruent triangles,
$\angle ASC = \angle PBC$.




Consider now the quadrilateral CBSD; this quadrilateral happens to be cyclic. This is because,

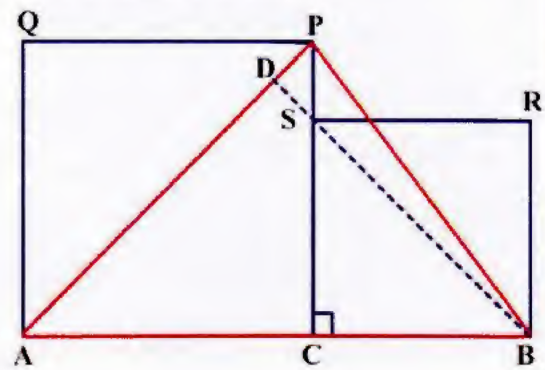
$$\angle DSC = \angle ASC$$
$$= \angle PBC = \angle DBC.$$

Hence $\angle BDS = \angle BCS = 90^o$. . It is now seen that SD $\perp$ BD which implies AS $\perp$ BD and hence AD is the altitude of $\Delta$APB. As altitudes (or their extensions pass through S, the point S is the orthocentre of $\Delta$APB.

**V.** Let us try to give a different proof using vector notation. We first recall in simple terms the notions of parallel and perpendicular vectors.

- Two vectors are perpendicular if their DOT PRODUCT is ZERO.
- Two vectors are parallel if they are "multiples" of each other.

Place the side AB of $\Delta$APB on $x$ axis with $|AC| = a$ units and $|BC| = b$ units where $a$ and $b$ are positive real numbers.




$\overline{BD} = k(\overline{BS})$ where $k$ is a positive real number.
$= k[b(\hat{i} - \hat{j})]$
since $\angle SBC = 45^o$ and $\overline{BS}$ subtends $135^o$ with positive $x$ direction.
$\overline{AP} = a(\hat{i} + \hat{j})$

Now, $\overline{AP}.\overline{BD} = [kb(\hat{i} - \hat{j})].[a(\hat{i} + \hat{j})] = kab(\hat{i} - \hat{j}).(\hat{i} + \hat{j}) = kab \times 0 = 0.$

$\Rightarrow$ $AP \perp BD$ ($\because$ AP and BD are neither parallel nor perpendicular to the axes).

This shows that BD is an altitude of $\Delta$APB.

Also, because $\angle PCA = 90^o$, $PC \perp AB$ and PC is an altitude of $\Delta$APB.

As altitudes PC and BD pass through S, we are sure now that S is the orthocentre of $\Delta$APB.

-oo000oo-

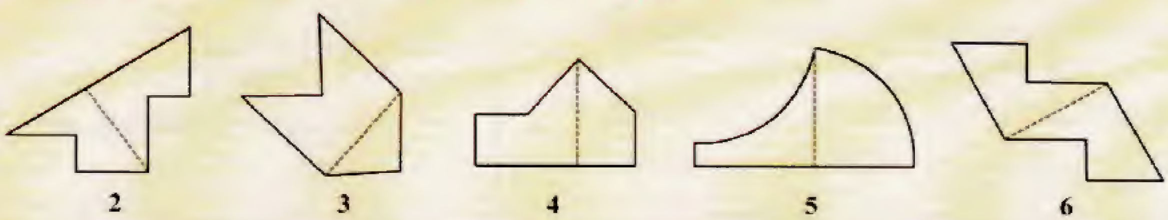
***Solution for the Challenge on page 17*** :

We disprove it. It is sufficient to disprove a conjecture with a single counter-example (as mentioned earlier in one of the previous articles).

There is a wonderful and startling connectivity between the 'lattice square area numbers' and sums of squares and other powers (on which topics numerous mathematicians like Brahmagupta, Fibonacci, Diophantus, Euler, Waring, Ramanujan and others have worked on different contexts).

65 is a lattice square area number which can be expressed as a sum of two squares in two different ways, viz., $65 = 1^2+8^2 = 4^2+7^2$. This serves as a simple counter example.

***Solution for "CAN YOU VISUALIZE?" on page 5.***




***A note on article SUBBIAH SIVASANKARANARAYANA PILLAI (S.S.PILLAI)***

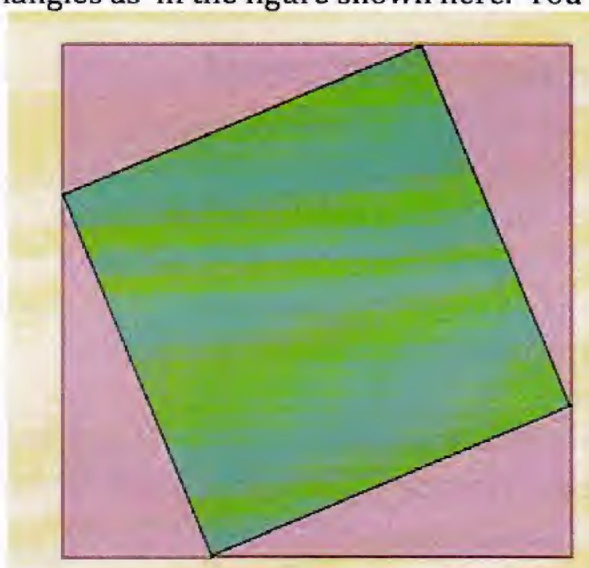
This is based on a piece of biographical note published in "***Indiarai valam varum Ganitha parisugal***" (in Tamil) authored by a passionate mathematics teacher ***Nallamur Kovi Pazhani***. The author has received quite a number of laurels for his work on popularization of mathematics, in particular in Tamilnadu. To know more of his work, he can be contacted on 94452 14218.

## *A CUT AND PASTE SOLUTION*

Suspose you are given a rectangle whose dimensions are not known. You want to find another rectangle whose area is double the area of the given triangle and whose perimeter is also twice the given rectangle's perimeter. (for example if you launch with a 12×5 rectangle, you want to get a 30×4 rectangle). Here is a method:

Have one more copy of the given rectangle; (remember that we need twice the area for the new rectangle). You can now think of dissecting each rectangle into two identical right-angled triangles. Thus you have four such triangles, all congruent.

Imagine arranging these four congruent triangles as in the figure shown here. You have a larger square in which there is a smaller vacant square (shown in green).



What can you say about the permeter of the outer square? It has exactly the perimeter length that is wanted for our new rectangle.

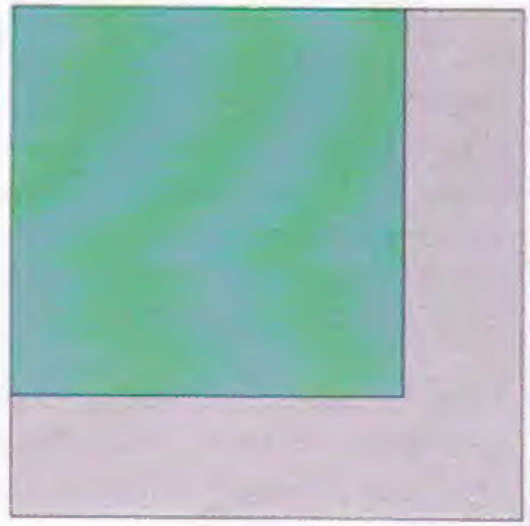
What can you conclude on the area of the four (shaded pink) triangles enclosing the inner square. Their total area is same as the area we need for the new rectangle.

(If you want to draw these figures, there may be some initial difficulty; begin with a rectangle having dimensions 12×5. Constructions will be easier. Do you see 'why'?)

We seem to need only the four pink triangles for constructing the required rectangle. We devise a strategy to make use of this idea.

Move the green square as shown in the following figure.

You have the green square at a corner and the remaining four trangles have to be sliced and adjusted to make the L-like shape.
The L-shape has the perimeter and area required.
All that you have to do now is to get the rectangle from the two pieces forming the L-shape.
Your required rectangle hence will look like this one in the last figure:



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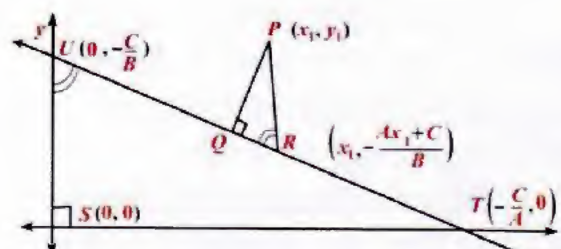
## A short sweet proof !

Here is a short and easy proof for the standard formula to find the distance from a point to a line.
*UT* is the given line $\boldsymbol{ax} + by + c = 0$ and P $(\boldsymbol{x_1}, \boldsymbol{y_1})$ is a given point.
You need the distance from P to the line.




Mark the coordinates of U and T as shown in the figure. PR is drawn parallel to the $\boldsymbol{y}$ axis. We have the following result from the idea of simila triangles *UST* and *PQR*.

$$\frac{PQ}{PR} = \frac{ST}{UT} \quad \Rightarrow \quad PQ = PR\left(\frac{ST}{UT}\right)$$

$$= \left|y_1 + \frac{Ax_1 + C}{B}\right| \left|\frac{C}{A}\right| \frac{1}{\sqrt{\frac{C^2}{A^2} + \frac{C^2}{B^2}}}$$

$$= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

(This was found in an old American journal. Is there a shorter one?)

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